per and lower bases 15 inches and 10 inches, respectively, and altitude 20 inches) at the rate of 10 cubic inches per second. When the depth is 8 inches at what rate is it increasing?

MECHANICS.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

14. Proposed by ALFRED HUME, C. E., Sc. D., Professor of Mathematics, University of Mississippi, University P. O., Miss.

"The center of a sphere of radius C moves in a circle of radius A and generates thereby a solid ring, as an anchor-ring: prove that the moment of inertia of this ring about an axis passing through the center of the direct circle and perpendicular to its plane is $\frac{1}{4}\pi^2\delta ac^2(4a^2+3c^2)$."

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

If the moment axis be the axis of z, the origin being the center of the ring, and the axis of x and y any two diameters at right angles the required moment could be obtained from $\int\int(x^2+y^2)z\,dx\,dy$, having the equation to the surface of the ring.

But the following method quoted by Williamson in the Int. Cal., New York Edition, 1884, art. 212 from Townsend is so concise I prefer to give it.

Let y, Y be the distances of any point in the meridian section of the sphere from that diameter of the section parallel to the moment axis, and to the moment axis. Then if dA be the element of area of the generating section, the mass of the elementary ring generated by dA is $2\pi\mu Y dA$, and the moment of inertia of this ring is $2\pi\mu Y^3 dA$.

$\therefore$ the required $M.I.=2\pi\mu\int Y^3 dA=2\pi\mu\int(a+y)^3 dA$

$$=2\pi\mu\int(a^3+3a^2y+3ay^2+y^3)dA\ldots.(1).$$

But from theory, $\int y\,dA=0$, $\int y^3 dA=0$, and if k be the radius of gyration of the generating section, $\int y^2 dA=Ak^2$: then (1) becomes

$$M.I.=2\pi\mu aA(a^2+3k^2)=2\pi^2\mu ac^2(a^2+\tfrac{3}{4}c^2)$$

$=\frac{\pi^2 \mu a c^2}{2}(4a^2+3c^2)\ldots(2)$, in which μ is used instead of δ, and the result is twice as great as given in the statement of the problem.

The advantage of this method lies in the fact that it is general for A and k^2, which are therefore the only quantities to be worked out before setting down the special result.

II. Solution by F. P. MATZ, M Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

According to well-known principles $V=\pi a^2\times 2\pi c=2\pi^2 ac^2$, $M=V\delta=2\pi^2\delta ac^2$, and the Radius of Gyration $=X=\sqrt{(\frac{1}{2}a^2+\frac{3}{8}c^2)}$. Hence the required Moment of Inertia, *Nystrom's Mechanics*, becomes $E=MX^2$ $=\frac{1}{4}\pi^2\delta ac^2(4a^2+3c^2)$.

III. Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics in the University of Mississippi.

If the center of the generating circle be taken as the origin and a perpendicular from this point to the axis of revolution as the axis of x, the equation of the moving circle is $x^2+y^2=c^2$.

Divide the ring formed into layers of infinitesimal thickness, dy, by planes parallel to the plane of the director circle.

The moment of inertia of any layer whose external radius is $a+x$ and internal $a-x$ is $\left[\frac{\pi}{2}\rho(a+x)^4-\frac{\pi}{2}\rho(a-x)^4\right]dy$, ρ being the density.

Therefore the moment of inertia of the entire ring is

$4\pi\rho a\int_{-c}^{c}(a^2+c^2-y^2)(c^2-y^2)^{\frac{1}{2}}dy$, substituting c^2-y^2 for x^2.

Performing the integration the result is $\frac{\pi^2\rho ac^2}{2}(4a^2+3c^2)$ which is double that given by Price.

This problem was also solved by *W. Wiggins, G. B. M. Zerr, and P. H. Philbrick*. Their solutions will be published next month.

PROBLEMS

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20. Proposed by CHAS. E. MYERS, Canton, Ohio.

A flexible cord of given length is suspended from two points whose co-ordinates are (x, y) and (x', y'). What must be the condition of the cord in order that it may hang in the arc of a circle?

21. Proposed by J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio.

Show that, in the wheel and axle, when a force P, acting at the circumference of the wheel, supports a weight Q upon the axle,